

Ministry of Education Malaysia

## Integrated Curriculum for Secondary Schools

CURRICULUM SPECIFICATIONS

# MATHEMATICS FORM 5 

Curriculum Development Centre Ministry of Education Malaysia 2006

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DECLARATION
OUR NATION, MALAYSIA, being dedicated

- to achieving a greater unity of all her peoples;
- to maintaining a democratic way of life;
- to creating a just society in which the wealth of the nation shall be equitably shared;
- to ensuring a liberal approach to her rich and diverse cultural traditions;
- to building a progressive society which shall be oriented to modern science and technology;

WE, her peoples, pledge our united efforts to attain these ends guided by these principles:

- BELIEF IN GOD
- LOYALTY TO KING AND COUNTRY
- UPHOLDING THE CONSTITUTION
- RULE OF LAW
- GOOD BEHAVIOUR AND MORALITY


## National Philosophy Edựation

Education in Malaysia is an ongoing effort towards further developing the potential of individuals in a holistic and integrated manner so as to produce individuals who are intellectually, spiritually, emotionally and physically balanced and harmonious, based on a firm belief in God. Such an effort is designed to produce Malaysian citizens who are knowledgeable and competent, who possess high moral standards, and who are responsible and capable of achieving a high level of personal well-being as well as being able to contribute to the betterment of the family, the society and the nation at large.

## PREFACE

Science and technology plays a critical role in realising Malaysia's aspiration to become a developed nation. Since mathematics is instrumental in the development of scientific and technological knowledge, the provision of quality mathematics education from an early age in the education process is thus important. The Malaysian school curriculum offers three mathematics education programs, namely Mathematics for primary schools, Mathematics and Additional Mathematics for secondary schools.

The Malaysian school mathematics curriculum aims to develop mathematical knowledge, competency and inculcate positive attitudes towards mathematics among pupils. Mathematics for secondary schools provides opportunities for pupils to acquire mathematical knowledge and skills, and develop higher order problem solving and decision making skills to enable pupils to cope with daily life challenges. As with other subjects in the secondary school curriculum, Mathematics aims to inculcate noble values and love for the nation in the development of a holistic person, who in turn will be able to contribute to the harmony and prosperity of the nation and its people.

Beginning 2003, English is used as the medium of instruction for Science and Mathematics subjects. The policy to change the medium of instruction for Science and Mathematics subjects follows a phased implementation schedule and is expected to be completed by 2008.

In the teaching and learning of Mathematics, the use of technology especially ICT is greatly emphasised. Mathematics taught in English, coupled with the use of ICT, provide greater opportunities for pupils to improve their knowledge and skills in mathematics because of the richness of resources and repositories of knowledge in English. Pupils will be better able to interact with pupils from other countries, improve their proficiency
in English and thus make the learning of mathematics more interesting and exciting.

The development of this Mathematics syllabus is the work of many individuals and experts in the field. On behalf of the Curriculum Development Centre, I would like to express much gratitude and appreciation to those who have contributed in one way or another towards this initiative

## hotpan

(MAHZAN BIN BAKAR SMP, AMP)
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## INTRODUCTION

A well-informed and knowledgeable society well versed in the use of mathematics to cope with daily life challenges is integral to realising the nation's aspiration to become an industrialised nation. Thus, efforts are taken to ensure a society that assimilates mathematics into their daily lives. Pupils are nurtured from an early age with the skills to solve problems and communicate mathematically, to enable them to make effective decisions.

Mathematics is essential in preparing a workforce capable of meeting the demands of a progressive nation. As such, this field assumes its role as the driving force behind various developments in science and technology. In line with the nation's objective to create a knowledge-based economy, the skills of Research \& Development in mathematics is nurtured and developed at school level.

As a field of study, Mathematics trains the mind to think logically and systematically in solving problems and making decisions. This discipline encourages meaningful learning and challenges the mind, and hence contributes to the holistic development of the individual. To this end, strategies to solve problems are widely used in the teaching and learning of mathematics. The development of mathematical reasoning is believed to be closely linked to the intellectual development and communication ability of pupils. Hence, mathematics reasoning skills are also incorporated in the mathematics activities to enable pupils to recognize, build and evaluate mathematics conjectures and statements.

In keeping with the National Education Philosophy, the Mathematics curriculum provides opportunities to pupils from various backgrounds and levels of abilities to acquire mathematical skills and knowledge. Pupils are then able to seek relevant information, and be creative in formulating alternatives and solutions when faced with challenges.

The general Mathematics curriculum has often been seen to comprise of discrete areas related to counting, measurement, geometry, algebra and solving problems. To avoid the areas to be continually seen as separate and pupils acquiring concepts and skills in isolation, mathematics is linked to everyday life and experiences in and out of school. Pupils will have the opportunity to apply mathematics in different contexts, and see the relevance of mathematics in daily life.

In giving opinions and solving problems either orally or in writing, pupils are guided in the correct usage of language and mathematics registers. Pupils are trained to select information presented in mathematical and nonmathematical language; interpret and represent information in tables, graphs, diagrams, equations or inequalities; and subsequently present information clearly and precisely, without any deviation from the original meaning.

Technology in education supports the mastery and achievement of the desired learning outcomes. Technology used in the teaching and learning of Mathematics, for example calculators, are to be regarded as tools to enhance the teaching and learning process and not to replace teachers.

Importance is also placed on the appreciation of the inherent beauty of mathematics. Acquainting pupils with the life-history of well-known mathematicians or events, the information of which is easily available from the Internet for example, will go a long way in motivating pupils to appreciate mathematics.

The intrinsic values of mathematics namely thinking systematically, accurately, thoroughly, diligently and with confidence, infused throughout the teaching and learning process; contribute to the moulding of character and the inculcation of positive attitudes towards mathematics. Together with these, moral values are also introduced in context throughout the teaching and learning of mathematics.

Assessment, in the form of tests and examinations helps to gauge pupils' achievements. The use of good assessment data from a variety of sources also provides valuable information on the development and progress of pupils. On-going assessment built into the daily lessons allows the identification of pupils' strengths and weaknesses, and effectiveness of the instructional activities. Information gained from responses to questions, group work results, and homework helps in improving the teaching process, and hence enables the provision of effectively aimed lessons.

## AIM

The mathematics curriculum for secondary schools aims to develop individuals who are able to think mathematically, and apply mathematical knowledge effectively and responsibly in solving problems and making decisions; and face the challenges in everyday life brought about by the advancement of science and technology.

## OBJ ECTIVES

The mathematics curriculum for the secondary school enables pupils to:
1 understand definitions, concepts, laws, principles, and theorems related to Number, Shape and Space, and Relationship;
2 widen the use of basic operations of addition, subtraction, multiplication and division related to Number, Shape and Space, and Relationship;

3 acquire basic mathematical skills such as:

- making estimation and rounding;
- measuring and constructing;
- collecting and handling data;
- representing and interpreting data;
- recognising and representing relationship mathematically;
- using algorithm and relationship;
- solving problems; and
- making decisions.

4 communicate mathematically;
5 apply knowledge and skills of mathematics in solving problems and making decisions;
6 relate mathematics with other areas of knowledge;
7 use suitable technologies in concept building, acquiring skills, solving problems and exploring the field of mathematics;
8 acquire mathematical knowledge and develop skills effectively and use them responsibly;
inculcate a positive attitude towards mathematics; and appreciate the importance and beauty of mathematics.

## CONTENT ORGANISATION

The Mathematics Curriculum content at the secondary school level is organised into three main areas, namely: Number; Shape and Space; and Relationship. Mathematical concepts related to the respective area, are further organised into topics. These topics are arranged in a hierarchical manner such that the more basic and tangible concepts appear first and the more complex and abstract concepts appear subsequently.

The Learning Area outlines the scope of mathematical knowledge, abilities and attitudes to be developed in pupils when learning the subject. They are developed according to the appropriate learning objectives and represented in five columns, as follows:

Column 1: Learning Objectives
Column 2: Suggested Teaching and Learning Activities
Column 3: Learning Outcomes
Column 4 : Points To Note; and
Column 5: Vocabulary.
The Learning Objectives define clearly what should be taught. They cover all aspects of the Mathematics curriculum programme and are presented in a developmental sequence designed to support pupils understanding of the concepts and skill of mathematics.
The Suggested Teaching and Learning Activities lists some examples of teaching and learning activities including methods, techniques, strategies and resources pertaining to the specific concepts or skills. These are, however, not the only intended approaches to be used in the classrooms. Teachers are encouraged to look for other examples, determine teaching and learning strategies most suitable for their students and provide appropriate teaching and learning materials. Teachers should also make cross-references to other resources such as the textbooks and the Internet.
The Learning Outcomes define specifically what pupils should be able to do. They prescribe the knowledge, skills or mathematical processes and values that should be inculcated and developed at the appropriate level. These behavioural objectives are measurable in all aspects.
In the Points To Note column, attention is drawn to the more significant aspects of mathematical concepts and skills. These emphases are to be taken into account so as to ensure that the concepts and skills are taught and learnt effectively as intended.

The Vocabulary consists of standard mathematical terms, instructional words or phrases which are relevant in structuring activities, in asking questions or setting tasks. It is important to pay careful attention to the use of correct terminology and these need to be systematically introduced to pupils in various contexts so as to enable them to understand their meaning and learn to use them appropriately.

## EMPHASES IN TEACHING AND LEARNING

This Mathematics Curriculum is arranged in such a way so as to give flexibility to teachers to implement an enjoyable, meaningful, useful and challenging teaching and learning environment. At the same time, it is important to ensure that pupils show progression in acquiring the mathematical concepts and skills.

In determining the change to another learning area or topic, the following have to be taken into consideration:

- The skills or concepts to be acquired in the learning area or in certain topics;
- Ensuring the hierarchy or relationship between learning areas or topics has been followed accordingly; and
- Ensuring the basic learning areas have been acquired fully before progressing to more abstract areas.
The teaching and learning processes emphasise concept building and skill acquisition as well as the inculcation of good and positive values. Besides these, there are other elements that have to be taken into account and infused in the teaching and learning processes in the classroom. The main elements focused in the teaching and learning of mathematics are as follows:


## 1. Problem Solving in Mathematics

Problem solving is the main focus in the teaching and learning of mathematics. Therefore the teaching and learning process must include problem solving skills which are comprehensive and cover the whole curriculum. The development of problem solving skills need to be emphasised so that pupils are able to solve various problems effectively. The skills involved are:

- Understanding the problem;
- Devising a plan;
- Carrying out the plan; and
- Looking back at the solutions.

Various strategies and steps are used to solve problems and these are expanded so as to be applicable in other learning areas. Through these activities, students can apply their conceptual understanding of mathematics and be confident when facing new or complex situations. Among the problem solving strategies that could be introduced are:

- Trying a simple case;
- Trial and improvement;
- Drawing diagrams;
- Identifying patterns;
- Making a table, chart or systematic list;
- Simulation;
- Using analogies;
- Working backwards;
- Logical reasoning; and
- Using algebra.


## 2. Communication in Mathematics

Communication is an essential means of sharing ideas and clarifying the understanding of Mathematics. Through communication, mathematical ideas become the object of reflection, discussion and modification. The process of analytical and systematic reasoning helps pupils to reinforce and strengthen their knowledge and understanding of mathematics to a deeper level. Through effective communication, pupils will become efficient in problem solving and be able to explain their conceptual understanding and mathematical skills to their peers and teachers.
Pupils who have developed the skills to communicate mathematically will become more inquisitive and, in the process, gain confidence. Communication skills in mathematics include reading and understanding problems, interpreting diagrams and graphs, using correct and concise mathematical terms during oral presentations and in writing. The skills should be expanded to include listening.

Communication in mathematics through the listening process occurs when individuals respond to what they hear and this encourages individuals to think using their mathematical knowledge in making decisions.

Communication in mathematics through the reading process takes place when an individual collects information and data and rearranges the relationship between ideas and concepts.
Communication in mathematics through the visualisation process takes place when an individual makes an observation, analyses, interprets and synthesises data and presents them in the form of geometric board, pictures and diagrams, tables and graphs. An effective communication environment can be created by taking into consideration the following methods:

- Identifying relevant contexts associated with environment and everyday life experience of pupils;
- Identifying pupils' interests;
- Identifying suitable teaching materials;
- Ensuring active learning;
- Stimulating meta-cognitive skills;
- . Inculcating positive attitudes; and
- . Setting up conducive learning environment.

Effective communication can be developed through the following methods:
Oral communication is an interactive process that involves psychomotor activities like listening, touching, observing, tasting and smelling. It is a twoway interaction that takes place between teacher and pupils, pupils and pupils, and pupils and object.

Some of the more effective and meaningful oral communication techniques in the learning of mathematics are as follows:

- Story-telling and question and answer sessions using one's own words;
- Asking and answering questions;
- Structured and unstructured interviews;
- Discussions during forums, seminars, debates and brainstorming sessions; and
- Presentation of findings of assignments.

Written communication is the process whereby mathematical ideas and information are disseminated through writing. The written work is usually the result of discussion, input from people and brainstorming activities when working on assignments. Through writing, pupils will be encouraged to think in depth about the mathematics content and observe the relationships between concepts. Examples of written communication activities that can be developed through assignments are:

- Doing exercises;
- Keeping journals;
- Keeping scrap books;
- Keeping folios;
- Keeping portfolios;
- Undertaking projects; and
- Doing written tests.

Representation is a process of analysing a mathematical problem and interpreting it from one mode to another. Mathematical representation enables pupils to find relationships between mathematical ideas that are informal, intuitive and abstract using everyday language. For example $6 x y$ can be interpreted as a rectangular area with sides $2 x$ and $3 y$. This will make pupils realise that some methods of representation are more effective and useful if they know how to use the elements of mathematical representation.

## 3. Reasoning in Mathematics

Logical Reasoning or thinking is the basis for understanding and solving mathematical problems. The development of mathematical reasoning is closely related to the intellectual and communicative development of pupils. Emphasis on logical thinking, during mathematical activities opens up pupils' minds to accept mathematics as a powerful tool in the world today.
Pupils are encouraged to estimate, predict and make intelligent guesses in the process of seeking solutions. Pupils at all levels have to be trained to investigate their predictions or guesses by using concrete material, calculators, computers, mathematical representation and others. Logical reasoning has to be absorbed in the teaching of mathematics so that pupils can recognise, construct and evaluate predictions and mathematical arguments.

## 4. Mathematical Connections

In the mathematics curriculum, opportunities for making connections must be created so that pupils can link conceptual to procedural knowledge and relate topics within mathematics and other learning areas in general.

The mathematics curriculum consists of several areas such as arithmetic, geometry, algebra, measures and problem solving. Without connections between these areas, pupils will have to learn and memorise too many concepts and skills separately. By making connections, pupils are able to see mathematics as an integrated whole rather than a jumble of unconnected ideas. When mathematical ideas and the curriculum are connected to real life within or outside the classroom, pupils will become more conscious of the importance and significance of mathematics. They will also be able to use mathematics contextually in different learning areas and in real life situations.

## 5. Application of Technology

The teaching and learning of mathematics should employ the latest technology to help pupils understand mathematical concepts in depth, meaningfully and precisely and enable them to explore mathematical ideas. The use of calculators, computers, educational software, websites in the Internet and relevant learning packages can help to upgrade the pedagogical approach and thus promote the understanding of mathematical concepts.
The use of these teaching resources will also help pupils absorb abstract ideas, be creative, feel confident and be able to work independently or in groups. Most of these resources are designed for self-access learning. Through self-access learning pupils will be able to access knowledge or skills and informations independently according to their own pace. This will
serve to stimulate pupils interests and develop a sense of responsibility towards their learning and understanding of mathematics.
Technology however does not replace the need for all pupils to learn and master the basic mathematical skills. Pupils must be able to efficiently add, subtract, multiply and divide without the use of calculators or other electronic tools. The use of technology must therefore emphasise the acquisition of mathematical concepts and knowledge rather than merely doing calculation.

## APPROACHES IN TEACHING AND LEARNING

The belief on how mathematics is learnt influence how mathematical concepts are to be taught. Whatever belief the teachers subscribe to, the fact remains that mathematical concepts are abstract. The use of teaching resources is therefore crucial in guiding pupils to develop matematical ideas. Teachers should use real or concrete materials to help pupils gain experience, construct abstract ideas, make inventions, build self confidence, encourage independence and inculcate the spirit of cooperation.
The teaching and learning materials used should contain self diagnostic elements so that pupils know how far they have understood the concepts and acquire the skills.
In order to assist pupils in having positive attitudes and personalities, the intrinsic mathematical values of accuracy, confidence and thinking systematically have to be infused into the teaching and learning process. Good moral values can be cultivated through suitable contexts. Learning in groups for example can help pupils to develop social skills, encourage cooperation and build self confidence. The element of patriotism should also be inculcated through the teaching and learning process in the classroom using certain topics.

Brief historical anecdotes related to aspects of mathematics and famous mathematicians associated with the learning areas are also incorporated into the curriculum. It should be presented at appropriate points where it provides students with a better understanding and appreciation of mathematics.
Various teaching strategies and approaches such as direct instruction, discovery learning, investigation, guided discovery or other methods must be incorporated. Amongst the approaches that can be given consideration include the following:

- Pupils-centered learning that is interesting;
- Different learning abilities and styles of pupils;
- Usage of relevant, suitable and effective teaching materials; and
- Formative evaluation to determine the effectiveness of teaching and learning.
The choice of an approach that is suitable will stimulate the teaching and learning environment inside or outside the classroom. Approaches that are considered suitable include the following:
- Cooperative learning;
- Contextual learning;
- Mastery learning;
- Constructivism;
- Enquiry-discovery; and
- Future studies.


## EVALUATION

Evaluation or assessment is part of the teaching and learning process to ascertain the strengths and weaknesses of pupils. It has to be planned and carried out as part of the classroom activities. Different methods of
assessment can be conducted. These maybe in the form of assignments, oral questioning and answering, observations and interviews. Based on the response, teachers can rectify pupils misconceptions and weaknesses and also improve their own teaching skills. Teachers can then take subsequent effective measures in conducting remedial and enrichment activities in upgrading pupils' performances.
LEARNING OBJ ECTIVES
Pupils will be taught to...
1 Understand and use the
concept of number in base
two, eight and five.

## SUGGESTED TEACHING AND

LEARNING ACTIVITIES
Use models such as a clock face or a counter which uses a particular number base.

Number base blocks of twos, eights and fives can be used to demonstrate the value of a number in the respective number bases.

For example:
$243_{5}$ is


2


4

Discuss:

- digits used,
- place values
in the number system with a particular number base.

LEARNING OUTCOMES
Pupils will be able to.
(i) State zero, one, two, three, ..., as a number in base:
a) two,
b) eight,
c) Five.
(ii) State the value of a digit of a number in base:
a) two,
b) eight,
c) Five.
(iii) Write a number in base:
a) two,
b) eight,
c) five
in expanded notation.

## POINTS TO NOTE

Emphasise the ways to read numbers in various bases.
Examples:

- $101_{2}$ is read as "one zero one base two"
- $7205_{8}$ is read as "seven two zero five base eight"
- $432_{5}$ is read as "four three two base five"

Numbers in base two are also known as binary numbers.

Examples of numbers in expanded notation:

- $10110_{2}=1 \times 2^{4}+0 \times 2^{3}$


## expanded

 notation
## notation

$+1 \times 2^{2}$
$+1 \times 2^{1}$
$+0 \times 2^{0}$

- $325_{8}=3 \times 8^{2}+2 \times 8^{1}$

$$
+5 \times 8^{0}
$$

- $3041_{5}=3 \times 5^{3}+0 \times 5^{2}$

$$
+4 \times 5^{1}+1 \times 5^{0}
$$



SUGGESTED TEACHING AND
LEARNING ACTIVITIES
Number base blocks of twos, eights and fives can also be used here. For example, to convert $10_{10}$ to a number in base two, use the least number of blocks $\left(2^{3}\right)$, tiles $\left(2^{2}\right)$, rectangles $\left(2^{1}\right)$ and squares $\left(2^{0}\right)$. Here, the least number of objects here are one block, no tiles, one rectangle and no squares. $\mathrm{So}, 10_{10}=1010_{2}$.

LEARNING OUTCOMES
Pupils will be able to.
(iv) Convert a number in base:
a) two,
b) eight,
c) five
to a number in base ten and vice versa.

POINTS TO NOTE

Perform repeated division to convert a number in base ten to a number in other bases.

For example, convert $714_{10}$ to a number in base five:
$5 \longdiv { 7 1 4 }$
5)142 --4
$5 \longdiv { 2 8 } - - 2$
$5 \longdiv { 5 } - - - 2$
$5 \longdiv { 1 } \quad 1 - - 0$
$50--1$
$\therefore 714_{10}=10324_{5}$

VOCABULARY
convert


Limit conversion of numbers to base two, eight and five only.


SUGGESTED TEACHING AND
LEARNING ACTIVITIES

As reinforcement, let pupils play a game; for example, matching cards of graphs with their respective functions. When the pupils have their matching partners, ask them to group themselves into four groups of types of functions. Finally, ask each group to name the type of function that is depicted on the cards.

## LEARNING OUTCOMES

Pupils will be able to.
(iii) Identify:
a) the shape of graph given a type of function,
b) the type of function given a graph,
c) the graph given a function and vice versa.
(iv) Sketch the graph of a given linear, quadratic, cubic or reciprocal function.
LEARNING OBJ ECTIVES
Pupils will be taught to...
2 Understand and use the
concept of the solution of
an equation by graphical
method.
3 Understand and use the
concept of the region
representing inequalities in
two variables.

## SUGGESTED TEACHING AND

LEARNING ACTIVITIES
Explore on the graphing calculator or the Geometer's Sketchpad to relate the $x$-coordinate of a point of intersection of two appropriate graphs to the solution of a given equation. Make generalisation about the point(s) of intersection of two graphs.

Discuss that if one point in a region satisfies $y>a x+b$ or $y<a x+b$, then all points in the region satisfy the same inequality.

LEARNING OUTCOMES
Pupils will be able to.
(i) Find the point(s) of intersection of two graphs.
(ii) Obtain the solution of an equation by finding the point(s) of intersection of two graphs.
(iii) Solve problems involving solution of an equation by graphical method.
(i) Determine whether a given point satisfies:
$y=a x+b$, or
$y>a x+b$, or
$y<a x+b$.
(ii) Determine the position of a given point relative to the graph $y=a x+b$.
(iii) Identify the region satisfying $y>a x+b$ or $y<a x+b$.

## POINTS TO NOTE

Use the traditional graph plotting exercise if the graphing calculator or the Sketchpad is unavailable.

Involve everyday problems.

For Learning Objective 3, include situations involving $x=a, x \geq a$, $x>a, x \leq a$ or $x<a$.
point of intersection

| LEARNING OBJ ECTIVES <br> Pupils will be taught to... | SUGGESTED TEACHING AND <br> LEARNING ACTIVITIES | LEARNING OUTCOMES <br> Pupils will be able to... |
| :--- | :--- | :--- |
|  | Use the Sketchpad or the graphing <br> calculator to explore points relative to a <br> graph to make generalisation about <br> regions satisfying the given inequalities. | (iv)Shade the regions representing <br> the inequalities: <br> ab $y>a x+b$ or $y<a x+b$, <br> b) $y \geq a x+b$ or $y \leq a x+b$. |

## POINTS TO NOTE

Emphasise that:

- for the region representing $y>a x+b$ or $y<a x+b$, the line $y=a x+b$ is drawn as a dashed line to indicate that all points on the line $y=a x+b$ are not in the region.
- for the region representing $y \geq a x+b$ or $y \leq a x+b$, the line $y=a x+b$ is drawn as a solid line to indicate that all points on the line $y=a x+b$ are in the region.
vocabulary
region
dashed line
solid line

| LEARNING OBJ ECTIVES |
| :--- |
| Pupils will be taught to... |
| 1 Understand and use the |
| concept of combination of |
| two transformations. |
|  |

SUGGESTED TEACHING AND
LEARNING ACTIVITIES
Relate to transformations in real life situations such as tessellation patterns on walls, ceilings or floors.

Explore combined transformation using Geometer's Sketchpad, graphing calculator or overhead projector and transparencies.

Investigate the characteristics of an object and its image under combined transformation.

LEARNING OUTCOMES
Pupils will be able to.
POINTS TO NOTE
Begin with a point, followed by a line and an object.

Limit isometric transformations to translations, reflections and rotations.

```
VOCABULARY
```

combined
transformation
equivalent


LEARNING OBJ ECTIVES
Pupils will be taught to...
$\mathbf{3}$ Perform addition and
subtraction on matrices.
$\mathbf{4}$ Perform multiplication
of a matrix by a number.
Pa

SUGGESTED TEACHING AND
LEARNING ACTIVITIES

Relate to real life situations such as keeping score of medal tally or points in sports.

Relate to real life situations such as in industrial productions.

LEARNING OUTCOMES
Pupils will be able to.

## POINTS TO NOTE

VOCABULARY
(i) Determine whether addition or subtraction can be performed on two given matrices.
(ii) Find the sum or the difference of two matrices.
(iii) Perform addition and subtraction on a few matrices.
(iv) Solve matrix equations involving addition and subtraction.
(i) Multiply a matrix by a number.
(ii) Express a given matrix as multiplication of a matrix by a number.
(iii) Perform calculation on matrices involving addition, subtraction and scalar multiplication.

Limit to matrices with no more than three rows and three columns.

Include finding values of matrix equation unknown elements.

Multiplying a matrix by a number is known as scalar multiplication.
scalar multiplication

|  |  |  |  |  | $\sum 10$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LEARNING OBJ ECTIVES Pupils will be taught to... | SUGGESTED TEACHING AND <br> LEARNING ACTIVITIES |  | RNING OUTCOMES ils will be able to... | POINTS TO NOTE | VOCABULARY |
| 5 Perform multiplication of two matrices. |  |  | Solve matrix equations involving addition, subtraction and scalar multiplication. | Include finding values of unknown elements. |  |
|  | Relate to real life situations such as finding the cost of a meal in a restaurant. |  | Determine whether two matrices can be multiplied and state the order of the product when the two matrices can be multiplied. |  | product |
|  | For matrices $\mathbf{A}$ and $\mathbf{B}$, discuss the relationship between $\mathbf{A B}$ and $\mathbf{B A}$. |  | Find the product of two matrices. | Limit to matrices with no more than 3 rows and 3 columns. |  |
|  |  |  | Solve matrix equations involving multiplication of two matrices. | Limit to two unknown elements. |  |
| 6 Understand and use the concept of identity matrix. | Begin with discussing the property of the number 1 as an identity for multiplication of numbers. <br> Discuss: <br> - an identity matrix is a square matrix, <br> - there is only one identity matrix for each order. |  | Determine whether a given matrix is an identity matrix by multiplying it to another matrix. <br> Write identity matrix of any order. | Identity matrix is usually denoted by I and is also known as unit matrix. | identity matrix unit matrix |


| LEARNING OBJ ECTIVES |
| :--- |
| Pupils will be taught to... |$|$

SUGGESTED TEACHING AND
LEARNING ACTIVITIES
Discuss the properties:

- $\mathbf{A I}=\mathbf{A}$,
- $\quad \mathrm{IA}=\mathrm{A}$.

Relate to property of multiplicative inverse of numbers.

For example:

$$
2 \times 2^{-1}=2^{-1} \times 2=1
$$

In this example, $2^{-1}$ is the multiplicative inverse of 2 and vice versa.


| LEARNING OBJ ECTIVES |
| :--- |
| Pupils will be taught to... |

SUGGESTED TEACHING AND LEARNING ACTIVITIES

Use the method of solving simultaneous linear equations to show that not all square matrices have inverse matrices. For example, ask pupils to try to find the inverse matrix of $\left(\begin{array}{ll}3 & 2 \\ 6 & 4\end{array}\right)$.

## LEARNING OUTCOMES <br> Pupils will be able to..

(ii) Find the inverse matrix of a $2 \times$ 2 matrix by using:
a) method of solving simultaneous linear equations,
b) formula.

## POINTS TO NOTE

Steps to find the inverse matrix:

- solving simultaneous linear equations
$\left(\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right)\left(\begin{array}{ll}p & q \\ r & s\end{array}\right)=\left(\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right)$
$p+2 r=1, q+2 s=0$
$3 p+44=0,3 q+4 s=1$
where $\left(\begin{array}{ll}p & q \\ r & s\end{array}\right)$ is the
inverse matrix;

LEARNING OBJ ECTIVES
SUGGESTED TEACHING AND LEARNING ACTIVITIES

## POINTS TO NOTE

Carry out operations leading to the formula.

Use matrices and their respective inverse matrices in the previous method to relate to the formula. Express each inverse matrix as a multiplication of a matrix by a number. Compare the scalar multiplication to the original matrix and discuss how the determinant is obtained.

Discuss the condition for the existence of inverse matrix.

- using formula

For $\mathbf{A}=\left(\begin{array}{ll}a & b \\ c & d\end{array}\right)$,
$\mathbf{A}^{-1}=\left(\begin{array}{cc}\frac{d}{a d-b c} & \frac{-b}{a d-b c} \\ \frac{-c}{a d-b c} & \frac{a}{a d-b c}\end{array}\right)$
or
$\mathbf{A}^{-1}=\frac{1}{a d-b c}\left(\begin{array}{cc}d & -b \\ -c & a\end{array}\right)$
when $a d-b c \neq 0$.
$a d-b c$ is known as the determinant of the matrix $\mathbf{A}$.
$\mathbf{A}^{-1}$ does not exist if the determinant is zero.

| LEARNING OBJ ECTIVES Pupils will be taught to... |
| :---: |
| 8 Solve simultaneous linear equations by using matrices. |

SUGGESTED TEACHING AND

## LEARNING ACTIVITIES

Relate to equal matrices by witing down the simultaneous equations as equal matrices first. For example:
Write $2 x+3 y=13$

$$
4 x-y=5
$$

as equal matrices:

$$
\binom{2 x+3 y}{4 x-y}=\binom{13}{5}
$$

which is then expressed as:

$$
\left(\begin{array}{cc}
2 & 3 \\
4 & -1
\end{array}\right)\binom{x}{y}=\binom{13}{5}
$$

Discuss why:

- the use of inverse matrix is necessary. Relate to solving linear equations of type $a x=b$,
- it is important to place the inverse matrix at the right place on both sides of the equation.
(i) Write down simultaneous linear equations in matrix form.
(ii) Find the matrix $\binom{p}{q}$ in
$\left(\begin{array}{ll}a & b \\ c & d\end{array}\right)\binom{p}{q}=\binom{h}{k}$
by using inverse matrix.


LEARNING OUTCOMES | Pupils will be able to... | POINTS TO NOTE |
| :--- | :--- |

(iii) Solve simultaneous linear equations by using the matrix method.
(iv) Solve problems involving matrices.

The matrix method uses inverse matrix to solve simultaneous linear equations.


| LEARNING OBJ ECTIVES |
| :--- |
| Pupils will be taught to... |
| 1 Understand and use the |
| concept of direct variation. |
|  |

SUGGESTED TEACHING AND
LEARNING ACTIVITIES
1 Understand and use the concept of direct variation.

LEARNING OUTCOMES
Pupils will be able to.
(i) State the changes in a quantity with respect to changes in another quantity, in everyday life situations involving direct variation.
(ii) Determine from given information whether a quantity varies directly as another quantity.
(iv) Find the value of a variable in a direct variation when sufficient information is given.
(v) Solve problems involving direct variation for the following cases:
$y \propto x ; y \propto x^{2} ; y \propto x^{3} ; y \propto x^{\frac{1}{2}}$.
technology. For example, the Law of Charles and Gay-Lussac (or Charles' Law), Hooke's Law and the simple pendulum.

For the cases $y \propto x^{n}, n=2,3, \frac{1}{2}$,
discuss the characteristics of the graphs of $y$ against $x^{n}$.
Discuss the characteristics of the graph of $y$ against $x$ when $y \propto x$.

Relate to other areas like science and .

## POINTS TO NOTE

$y$ varies directly as $x$ if and only if $\frac{y}{x}$ is a constant.

If $y$ varies directly as $x$, the relation is written as $y \propto x$.
For the cases $y \propto x^{n}$,
limit $n$ to 2, 3 and $\frac{1}{2}$.
If $y \propto x$, then $y=k x$ where $k$ is the constant of variation.

Using:

- $y=k x$; or
- $\frac{y_{1}}{x_{1}}=\frac{y_{2}}{x_{2}}$
to get the solution.
vocabulary
direct variation quantity
constant of variation variable

| LEARNING OBJ ECTIVES Pupils will be taught to... | SUGGESTED TEACHING AND LEARNING ACTIVITIES |  | RNING OUTCOMES ils will be able to... | POINTS TO NOTE | VOCABULARY |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 Understand and use the concept of inverse variation. |  |  | State the changes in a quantity with respect to changes in another quantity, in everyday life situations involving inverse variation. | $y$ varies inversely as $x$ if and only if $x y$ is a constant. | inverse variation |
|  | Discuss the form of the graph of $y$ against $\frac{1}{x}$ when $y \propto \frac{1}{x}$. <br> Relate to other areas like science and technology. For example, Boyle's Law. |  | Determine from given information whether a quantity varies inversely as another quantity. | If $y$ varies inversely as $x$, the relation is written as $y \propto \frac{1}{x}$. |  |
|  |  |  |  | For the cases $y \propto \frac{1}{x^{n}}$, |  |
|  |  |  |  | limit $n$ to 2,3 and $\frac{1}{2}$. |  |
|  |  | (iii) | Express an inverse variation in the form of equation involving two variables. | If $y \propto \frac{1}{x}$, then $y=\frac{k}{x}$ where $k$ is the constant of variation. |  |


| LEARNING OBJ ECTIVES <br> Pupils will be taught to... | SUGGESTED TEACHING AND <br> LEARNING ACTIVITIES | LEARNING OUTCOMES <br> Pupils will be able to... |
| :--- | :--- | :--- |
|  | (iv)Find the value of a variable in an <br> inverse variation when sufficient <br> information is given. <br>  <br>  <br>  <br>  <br> For the cases $y \propto \frac{1}{x^{n}}, n=2,3, \frac{1}{2}$, <br> discuss the characteristics of the graph of | (v)Solve problems involving <br> inverse variation for the <br> following cases: <br> $x^{n}$ <br> $y \propto \frac{1}{x} ; y \propto \frac{1}{x^{2}} ; y \propto \frac{1}{x^{3}} ; y \propto \frac{1}{x^{\frac{1}{2}}}$ |


| LEARNING OBJ ECTIVES Pupils will be taught to.. | SUGGESTED TEACHING AND LEARNING ACTIVITIES | LEARNING OUTCOMES Pupils will be able to... | POINTS TO NOTE | VOCABULARY |
| :---: | :---: | :---: | :---: | :---: |
| 3 Understand and use the concept of joint variation. | Discuss joint variation for the three cases in everyday life situations. <br> Relate to other areas like science and technology. <br> For example: <br> $I \propto \frac{V}{R}$ means the current $I$ varies directly as the voltage $V$ and varies inversely as the resistance $R$. | (i) Represent a joint variation by using the symbol $\propto$ for the following cases: <br> a) two direct variations, <br> b) two inverse variations, <br> c) a direct variation and an inverse variation. <br> (ii) Express a joint variation in the form of equation. <br> (iii) Find the value of a variable in a joint variation when sufficient information is given. <br> (iv) Solve problems involving joint variation | For the cases $y \propto x^{n} z^{n}$, $y \propto \frac{1}{x^{n} z^{n}}$ and $y \propto \frac{x^{n}}{z^{n}}$, limit $n$ to 2, 3 and $\frac{1}{2}$. | joint variation |





| LEARNING OBJ ECTIVES |
| :--- |
| Pupils will be taught to... |
| 1 Understand and use the |
| concept of probability of an |
| event. |

SUGGESTED TEACHING AND
LEARNING ACTIVITIES
Discuss equiprobable sample space through concrete activities and begin with simple cases such as tossing a fair coin.

In cases that can be seen as built up of simple cases, for example tossing a fair coin and a fair die, tree diagrams can be used to obtain the sample space.

Graphing calculator can be used for these activities.

Discuss events that produce $\mathrm{P}(A)=1$ and $\mathrm{P}(A)=0$.

LEARNING OUTCOMES
Pupils will be able to.
(i) Determine the sample space of an experiment with equally likely outcomes.
(ii) Determine the probability of an event with equiprobable sample space.
(iii) Solve problems involving probability of an event.

## POINTS TO NOTE

Limit to sample space with equally likely outcomes.

A sample space in which each event is equally likely is called equiprobable sample space.

The probability of an event $A$, with equiprobable sample space $S$, is $\mathrm{P}(A)=\frac{n(A)}{n(S)}$

Use tree diagram where appropriate.

Include everyday problems and making predictions.

VOCABULARY
equally likely
equiprobable sample space
tree diagram

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LEARNING OBJ ECTIVES Pupils will be taught to.. | SUGGESTED TEACHING AND LEARNING ACTIVITIES |  | RNING OUTCOMES ils will be able to... | POINTS TO NOTE | VOCABULARY |
| 2 Understand and use the concept of probability of the complement of an event. | Include events in real life situations such as winning or losing a game and passing or failing an exam. | (i) (ii) | State the complement of an event in: <br> a) words, <br> b) set notation. <br> Find the probability of the complement of an event. | The complement of an event $A$ is the set of all outcomes in the sample space that are not included in the outcomes of event $A$. | complement of an event |
| 3 Understand and use the concept of probability of combined event. | Use real life situations to show the relationship between <br> - $A$ or $B$ and $A \cup B$, <br> - $A$ and $B$ and $A \cap B$. <br> For example, being chosen to be a member of an exclusive club with restricted conditions. <br> Tree diagrams and coordinate planes are useful tools that can be used to find all the outcomes of combined events. | (i) (ii) | List the outcomes for events: <br> a) $A$ or $B$ as elements of the set $A \cup B$, <br> b) $A$ and $B$ as elements of the set $A \cap B$. <br> Find the probability by listing the outcomes of the combined event: <br> a) $A$ or $B$, <br> b) $A$ and $B$. |  | combined event |


| LEARNING OBJ ECTIVES Pupils will be taught to... | SUGGESTED TEACHING AND LEARNING ACTIVITIES |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Use two-way classification tables of events from newspaper articles or statistical data to find the probability of combined events. Ask students to create tree diagrams from these tables. <br> Example of a two-way classification table: |  |  |  |
|  |  | Means of going to work |  |  |
|  | Officers | Car | Bus | Others |
|  | Men | 56 | 25 | 83 |
|  | Women | 50 | 42 | 37 |

## Disscuss:

- situations where decisions have to be made based on probability, for example in business, such as determining the value for specific insurance policy and time slot for advertisements on television,
- the statement "probability is the underlying language of statistics".

LEARNING OUTCOMES Pupils will be able to...
(iii) Solve problems involving probability of combined event.

Emphasise that:

- knowledge about probability is useful in making decisions;
- prediction based on probability is not definite or absolute.



|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| LEARNING OBJ ECTIVES Pupils will be taught to... | SUGGESTED TEACHING AND LEARNING ACTIVITIES | LEARNING OUTCOMES Pupils will be able to... | POINTS TO NOTE | VOCABULARY |
| 1 Understand and use the concept of longitude. | Models such as globes should be used. <br> Introduce the meridian through <br> Greenwich in England as the Greenwich Meridian with longitude $0^{\circ}$. <br> Discuss that: <br> - all points on a meridian have the same longitude, <br> - there are two meridians on a great circle through both poles, <br> - meridian with longitudes $x^{\circ} \mathrm{E}$ (or W) and $\left(180^{\circ}-x^{\circ}\right) \mathrm{W}($ or E$)$ form a great circle through both poles. | (i) Sketch a great circle through the North and South Poles. <br> (ii) State the longitude of a given point. <br> (iii) Sketch and label a meridian with the longitude given. <br> (iv) Find the difference between two longitudes. | Emphasise that longitude $180^{\circ} \mathrm{E}$ and longitude $180^{\circ} \mathrm{W}$ refer to the same meridian. <br> Express the difference as an angle less than or equal to $180^{\circ}$. | great circle <br> meridian <br> longitude |
| 2 Understand and use the concept of latitude. |  | (i) Sketch a circle parallel to the equator. <br> (ii) State the latitude of a given point. | Emphasise that <br> - the latitude of the equator is $0^{\circ}$; <br> - latitude ranges from $0^{\circ}$ to $90^{\circ} \mathrm{N}$ (or S). | equator <br> latitude |


LEARNING OBJ ECTIVES
Pupils will be taught to...
4 Understand and use the
concept of distance on the
surface of the earth to solve
problems.

Find the distance between two cities or towns on the same meridian, as a group project.

Sketch the angle at the centre of the earth that is subtended by the arc between two given points along the equator. Discuss how to find the value of this angle.

LEARNING OUTCOMES
SUGGESTED TEACHING AND
LEARNING ACTIVITIES
(i) Find the length of an arc of a great circle in nautical mile, given the subtended angle at the centre of the earth and vice versa.
(ii) Find the distance between two points measured along a meridian, given the latitudes of both points;.
(iii) Find the latitude of a point given the latitude of another point and the distance between the two points along the same meridian.
(iv) Find the distance between two points measured along the equator, given the longitudes of both points.
(v) Find the longitude of a point given the longitude of another point and the distance between the two points along the equator.

POINTS TO NOTE
Limit to nautical mile as the unit for distance.

Explain one nautical mile as the length of the arc of a great circle subtending a one minute angle at the centre of the earth.

VOCABULARY
nautical mile

| 凸边 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| LEARNING OBJ ECTIVES Pupils will be taught to... | SUGGESTED TEACHING AND LEARNING ACTIVITIES | LEARNING OUTCOMES Pupils will be able to... | POINTS TO NOTE | VOCABULARY |
|  | Use models. | (vi) State the relation between the radius of the earth and the radius of a parallel of latitude. <br> (vii) State the relation between the length of an arc on the equator between two meridians and the length of the corresponding arc on a parallel of latitude. |  |  |
|  | Find the distance between two cities or towns on the same parallel of latitude as a group project. | (viii) Find the distance between two points measured along a parallel of latitude. |  |  |
|  |  | (ix) Find the longitude of a point given the longitude of another point and the distance between the two points along a parallel of latitude. |  |  |
|  | Use the globe and a few pieces of string to show how to determine the shortest distance between two points on the surface of the earth. | (x) Find the shortest distance between two points on the surface of the earth. | Limit to two points on the equator or a great circle through the poles. |  |
|  |  | (xi) Solve problems involving: <br> a) distance between two points, <br> b) travelling on the surface of the earth. | Use knot as the unit for speed in navigation and aviation. |  |





| LEARNING OBJ ECTIVES Pupils will be taught to... | SUGGESTED TEACHING AND <br> LEARNING ACTIVITIES | LEARNING OUTCOMES Pupils will be able to... |  | POINTS TO NOTE | VOCABULARY |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Carry out group project: | (iii) | Draw: |  |  |
|  | Draw plan and elevations of buildings or |  | a) the plan, |  |  |
|  | structures, for example pupils or teacher's dream homes and construct a |  | b) the front elevation, |  |  |
|  | scale model based on the drawings. |  | c) the side elevation |  |  |
|  |  |  | of a solid object to scale. |  |  |
|  | Involve real life situations such as in building prototypes and using actual home plans. | (iv) | Solve problems involving plan and elevation. |  |  |

